as the 30-deg case and, therefore, the source point is closer to the inlet resulting in a larger pressure gradient for the same M.) Therefore, I extended the stability analysis and found that instead of inviscid, diverging, subsonic flow being unstable, it was conditionally unstable depending on the size of the pressure gradient. The fact that the u, v, and ρ, and p_T, T_T and θ boundary conditions are unstable at different pressure gradients may be because they model different upstream conditions.

In a personal conversation, Moretti stated he had solved the higher Mach number and divergence angle flows and found them to be stable. Because our numerical procedures appear to be similar (provided I use the p_T, T_T, and θ boundary conditions), I cannot explain this discrepancy. Moretti feels this is due to an error in my code.

Therefore, based on my work (p_T, T_T, and θ calculations; u, v, and ρ calculations; supersonic inflow/outflow calculations; 1-D stability analysis) inviscid, diverging, subsonic flow is conditionally unstable. Moretti argues that based on his work, these flows are stable (recall he discounts my u, v, and ρ calculations and 1-D stability analysis). Considering that my conclusions, as well as Moretti's, are based on numerical computations, there is a considerable margin for error.

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Marangoni-Number-Dependent Bubble Velocity

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A RECENT article by Papazian et al.[1] concerning bubble behavior during solidification in low gravity. One part of their paper deals with the effects of an imposed temperature gradient on bubble motion in the melt. A temperature gradient parallel to the phase boundary of two fluids induces a gradient of interfacial tension which generates convection. Young et al.[2] have shown that in the case of fluid particles in a fluid medium the interfacial tension driven convection can cause linear particle motions. Papazian et al.[1,3] applied the formula of Young et al. for estimates of particle velocities in their space experiments. Their experiments, however, yielded bubble velocities that were much lower than calculated (<1%). The authors discussed possible explanations for this discrepancy and finally concluded that "reasonable doubts exist as to whether thermocapillary forces will cause bubble motion in low gravity."[1] We think that this statement calls for a comment: Aside from the influence of contamination of the phase boundary (which has been thoroughly tested by the authors[1]), a reason for the discrepancies relates to the magnitude of the Marangoni numbers, which in the experiments of Papazian et al. were between 1 and 5000.[4] In connection with Marangoni numbers the authors mention the criterion of Pearson[5] which says that a critical value of $Ma_c = 80$ must be exceeded for convection cells to be induced in a horizontal geometry. In such a situation sufficiently large Ma are required to maintain original variations in surface temperature. However, the application of Pearson's criterion in connection with Young's theory of particle motion can be misleading. The following considerations will show that low Marangoni numbers, as in the experiments of Young et al.[2] and Coriell et al.[6] ($10^{-4} < Ma < 10^{-2}$), are prerequisite for Young's calculation to be valid.

The particle velocity according to Young et al.[2] is:

$$v_p = -[2R/(6\eta_e + 9\eta_i] \text{ grad}\gamma \tag{1}$$

with gradγ = constant, and η_e, η_i are the viscosity of medium and particle, respectively, R is the radius, and γ the interfacial tension.

When a constant gradient of temperature is imposed and the temperature distribution depends on conduction only the velocity is:

$$v_p = [2R\kappa_e/(2\eta_e + 3\eta_i)(2\kappa_e + \kappa_i)](\partial\gamma/\partial T)\text{grad}_0 T \tag{2}$$

where κ is heat conductivity, and grad$_0 T$ is the imposed gradient.

For bubbles in a surrounding liquid we have $\eta_e \gg \eta_i$ and $\kappa_e \gg \kappa_i$ and the velocity is:

$$v_p = -(R/2\eta_e)(\partial\gamma/\partial T)\text{grad}_0 T \tag{3}$$

But as soon as interfacial tension driven convection sets in the temperature distribution is distorted, since convective heat transport must also be taken into account. Under stationary conditions the temperature distribution is now determined by:

$$\vec{v} \text{ grad } T = \chi \Delta T \tag{4}$$

where χ is the thermal diffusivity.

The ratio of convective and conductive heat transport is measured by the Péclet-number ($P_e = vR/\chi$).

For $Pe \to 0$, the left-hand side of Eq. (4) vanishes. This however, is one of the basic assumptions underlying the derivation of Eqs. (2) and (3), respectively. Taking into account that the flow velocity v involved in Young's model is of the same order as the particle velocity v_p, the Péclet number for interfacial tension driven convection may be identified with the Marangoni number:

$$Ma = [R^2(\partial\gamma/\partial T)\text{grad } T]/(\eta\chi)$$

In conclusion, it can be stated that bubble velocities calculated with Young's formula are in good agreement with observations provided that the Péclet number is small ($<10^{-2}$). In the experiments of Papazian et al. the Marangoni number, and by consequence also Pe, was higher by at least a factor of 100, so that only substantially reduced velocities (compared to

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Young et al. and Coriell et al.) were to be expected. Quantitative estimates of bubble velocities for $Pe > 1$ would require numerical calculations, but presently no such analysis seems to be available.

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Reply by Authors to H. Klein and A. Bewersdoff

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IN their Comment, Klein and Bewersdorff point out that the Young, Goldstein, and Block[1] formula gives a valid estimate of bubble velocities only when $Ma \ll 1$. We agree with this observation and have not suggested otherwise.[2,3] However, although the expected bubble velocities for $Ma > 1$ will be reduced compared to the Young et al. estimate, they will still *increase* as Ma increases. Thus, we maintain that the orders of magnitude discrepancy between the expected bubble velocities and our observed bubble immobility is signifiicant. In later papers, we described experiments and calculations designed to further investigate this discrepancy, and we were able to identify contamination as the most probable cause for the immobility of bubbles in our low-gravity experiments.[4,5] Well controlled experiments are still required in order to establish the precise nature of thermocapillary bubble motion in a low-gravity environment and its dependence on material properties. In addition, research on bubble migration velocity for $Ma > 1$ is currently in progress.[6]

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Errata: An Approach for Estimating Vibration Characteristics of Nonuniform Rotor Blades

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THE data for Illustrative Examples on page 998 should read as follows:

$$EI_l(x)\ddagger = \begin{cases} 2.5\times 10^7 \text{ lb in.}^2,\ 0\le x\le 0.2R \\ (4.332005 - 15.366799x + 26.696032x^2 \\ -15.153439x^3)\times 10^7 \text{ lb in.}^2,\ 0.2R < x \le R \end{cases}$$

$$m(x)\S = \begin{cases} 0.397549 + 93.7898x - 462.665x^2 \text{ lb}_\text{m}/\text{in.}, \\ 0\le x\le 0.2R\ 1.101767 - 0.512333x \text{ lb}_\text{m}/\text{in.}, \\ 0.2R < x \le R \end{cases}$$

In the metric system, these become

$$EI_l(x) = \begin{cases} 7.174175\times 10^4 \text{ Nm}^2,\ 0\le x\le 0.2R \\ (12.431425 - 44.097642x + 76.608802x^2 \\ 43.485369x^3)\times 10^4 \text{ Nm}^2,\ 0.2R < x \le R \end{cases}$$

$$m(x) = \begin{cases} 7.09943 + 1674.8982x - 8262.2716x^2 \text{ kg/m}, \\ 0\le x\le 0.2R\ 19.675355 - 9.149243x \text{ kg/m}, \\ 0.2R < x \le R \end{cases}$$

The errors are only of a typographical nature, and therefore, they do not affect the results presented in the published paper.

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